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Anomalous dimensions in gauge theories from rotating strings in $AdS_5 \times S^5$

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Abstract: Semi-classical soliton solutions for superstrings in $AdS_5 \times S^5$ are used to predict the dimension of gauge theory operators in $\mathcal{N} = 4\,\mathrm{SU}(N)$ SYM theory. We discuss the possible origin of scaling violations on the gauge theory side.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Understanding the AdS/CFT correspondence [1, 2, 3] to full string theory level can lead to important insights about the $1/N$ expansion and strong coupling physics in Yang-Mills theories. It is therefore of interest to develop methods to study string propagation in backgrounds with R-R gauge fields. Recently, some direct checks of the AdS/CFT correspondence beyond the supergravity level were performed by considering a special sector of states where some global quantum numbers (such as R-charge or spin) are large [4]–[7]. The idea of [4] is based on the observation [8, 4, 9, 10] that string theory in certain [11, 12] R-R plane-wave backgrounds is solvable. Extensions in different directions were carried out in [13]–[29].

The strategy of [6], which was further elaborated in [30], is to identify certain semi-classical soliton solutions representing highly excited string states with gauge theory operators of some finite anomalous dimension. The classical energy in global AdS coordinates is identified with the conformal dimension of the corresponding state in the dual gauge theory.

The soliton solutions investigated in [6] can be separated into three classes:

i) Strings spinning on AdS_5, stretched along the radial direction ρ, at fixed angles on S^5 (an earlier study of this classical string solution is in [31]). They represent string states with spin S, and they have an energy

$$E \cong S + \frac{\sqrt{\lambda}}{\pi} \log \frac{S}{\sqrt{\lambda}} \,, \qquad S \gg \sqrt{\lambda} \,. \tag{1.1}$$

ii) Strings spinning on S^5, stretched along the radial direction ρ, representing string states carrying R-charge J. For them

$$E = J + \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{\lambda n^2}{J^2}} \,, \qquad J \gg \sqrt{\lambda} \,, \tag{1.2}$$

where the second term arises by considering small oscillations around the soliton state. This is the same formula obtained by [4] in the exactly solvable [8], [9] pp-wave background [11, 12].

JHEP06(2002)038

iii) Strings spinning on S^5 (i.e. with R-charge J), sitting at $\rho = 0$ and strectched along an angular direction. Here one obtains [6]

$$E \cong J + \frac{2\sqrt{\lambda}}{\pi} . \tag{1.3}$$

In [30], a more general solution interpolating between the cases i) and ii) was investigated, leading to a general formula for the energy as a function of S and J. Here we shall present a solution interpolating between the three cases i), ii), iii) described above. This will lead to a more general formula $E = E(S, J)$, which will reduce to the previously known cases by taking different limits. The string state (and thus the corresponding gauge operator) is not the same as the one studied in [30]. In a large J limit, the energy approaches the energy of the state of [30]. The main virtue of the present approach is to incorporate the features of the formulas (1.1)–(1.3) into a single general expression for the conformal dimension.

2. More general soliton rotating in AdS$_5$ and S^5

We consider type-IIB superstrings moving in the $AdS_5 \times S^5$ background, with metric:

$$\begin{aligned} ds^2 &= \alpha' R^2 \left[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\tilde{\Omega}_3^2 + d\psi^2 \sin^2 \theta + d\theta^2 + \cos^2 \theta d\Omega_3^2 \right] , \\ d\tilde{\Omega}_3^2 &= \cos^2 \beta d\tilde{\phi}^2 + d\beta^2 + \sin^2 \beta d\phi^2 , \\ d\Omega_3^2 &= \cos^2 \psi_1 d\psi_2^2 + d\psi_1^2 + \sin^2 \psi_1 d\psi_3^2 , \end{aligned} \tag{2.1}$$

where $R^2 = \sqrt{\lambda}$ and $\lambda = g_{\rm YM}^2 N$ is the 't Hooft coupling. We look for a soliton solution of the following form:

$$t = \kappa\tau , \qquad \phi = \omega\tau , \qquad \psi = \nu\tau , \tag{2.2}$$

$$\begin{aligned} \rho &= \rho(\sigma) = \rho(\sigma + 2\pi) , \qquad \theta = \theta(\sigma) = \theta(\sigma + 2\pi) , \\ \beta &= \frac{\pi}{2} , \qquad \psi_1 = \psi_2 = \psi_3 = \tilde{\phi} = 0 . \end{aligned} \tag{2.3}$$

It describes a string rotating in AdS$_5$ and in S^5 with independent angular velocity parameters ω and ν, which is stretched along the radial coordinate and along the angular coordinate θ of S^5. We will find below that this rotating soliton generalizes the solutions discussed in [6, 30], and smoothly interpolates beween all previously known cases.

The equations of motion and constraints become

$$\rho'' + \left(\omega^2 - \kappa^2\right) \cosh \rho \sinh \rho = 0 , \tag{2.4}$$

$$\theta'' + \nu^2 \cos\theta \sin\theta = 0 , \tag{2.5}$$

$$-\kappa^2 \cosh^2 \rho + \rho'^2 + \omega^2 \sinh^2 \rho + \theta'^2 + \nu^2 \sin^2 \theta = 0 , \tag{2.6}$$

where prime denotes derivative with respect to σ. The general solution is given by

$$\rho'^2 = \kappa^2 \cos^2 \alpha_0 - \left(\omega^2 - \kappa^2\right) \sinh^2 \rho , \tag{2.7}$$

$$\theta'^2 = -\nu^2 \sin^2 \theta + \kappa^2 \sin^2 \alpha_0 . \tag{2.8}$$

where α_0 is an integration constant.

We have chosen a convenient parametrization in terms of $\cos\alpha_0, \sin\alpha_0$ to take into account automatically that the solution describes a finite closed string stretching from $\rho = 0$ up to some $\rho = \rho_{\max}$, which is finite provided $\omega > \kappa$. The string is folded onto itself, and the interval $0 \leq \sigma < 2\pi$ is split into four segments. The first segment starts with $\rho = 0$ at $\sigma = 0$ up to $\rho_{\max}$ at $\sigma = \pi/2$.

Let us define parameters

$$a \equiv \frac{\nu^2}{\kappa^2 \sin^2\alpha_0}\,, \qquad b \equiv \frac{\omega^2 - \kappa^2}{\kappa^2\cos^2\alpha_0}\,. \tag{2.9}$$

We will assume $b > 0$. There are different situations according to the value of a.

The case $a = 1$ gives a solution with infinite energy, unless $\theta \equiv \pi/2$, which is the case discussed by [30]. This includes the cases discussed in sects. 2 and 3 of [6], corresponding to the pp wave limit and to the Regge string.

If $a < 1$, then $\theta \in [0, \pi]$. This describes a closed string stretched around the great circle of S^5. This solution is a generalization of a similar solution (but with $\rho \equiv 0$) discussed in [6] (it reduces to that particular constant ρ case when $b \gg 1$). As pointed out in [6], the solution seems unstable, since a small perturbation of the string makes it to slide over the sides of the sphere due to the string tension.

Here we shall consider the interesting case of $a > 1$. Then $\theta \in [0, \theta_{\max}]$, where $\theta_{\max} = \arcsin(a^{-1/2})$, $\theta_{\max} < \pi/2$. It generalizes the case discussed in section 4.1 of [6] (corresponding to the special case $\rho \equiv 0$, $a > 1$) to strings which also stretch along the radial direction. In the regime $a \sim 1$, the major contribution to the quantum numbers of the string will come from the region $\theta \sim \pi/2$. For this reason, in this regime we will recover the results of [30] for the anomalous dimension, with a small correction due to the fact that the string is also stretched along the θ direction.

Thus there is a general solution interpolating between the different known cases. This will give a more general formula relating the dimension of the operators with the R-charge and spin.

Note that there are no solutions in this class where $\theta \in [\pi/2, \pi/2 + \epsilon]$. Either θ is stuck at $\pi/2$, or it goes all the way up to $\theta = 0$. However, one can consider time-dependent small fluctuations around $\theta = \pi/2$. This semiclassical quantization was carried out in [6], [30], and it leads to the full spectrum (1.2), including the contributions of oscillators. For the present soliton, the semiclassical quantization in terms of a small θ expansion, retaining only the harmonic oscillations, is meaningful only for small J, where $\theta_{\max}$ is small, but not in the interesting region of large J (corresponding to $a \sim 1$), where the angular variable θ takes all values from 0 to $\pi/2$.

From eqs. (2.7), (2.8), we obtain

$$(\kappa\sin\alpha_0)\ \sigma = \int_0^\theta d\theta' \frac{1}{\sqrt{1 - a\sin^2\theta'}} = \mathcal{F}(\theta, a)\,, \tag{2.10}$$

$$(\kappa\cos\alpha_0)\ \sigma = \int_0^\rho d\rho' \frac{1}{\sqrt{1 - b\sinh^2\rho'}} = -i\mathcal{F}(i\rho, -b)\,, \tag{2.11}$$

where $\mathcal{F}(x, m)$ represents the elliptic integral of the first kind. These equations define $\rho = \rho(\theta)$.

In the present case of $a > 1$ and $b > 0$, there are maximum values of θ and ρ taken by the string which are given by $\theta_{\rm max} = \arcsin(a^{-1/2})$, $\rho_{\rm max} = {\rm arcsinh}(b^{-1/2})$. From now on it is convenient to trade the parameters ω, ν by a, b (see (2.9)).

The periodicity condition (2.3) determines α_0, κ in terms of a, b. As a function of σ, $\rho(\sigma)$, $\theta(\sigma)$ start at $\rho = 0$, $\theta = 0$ at $\sigma = 0$ and reach a maximum value of ρ and θ at the point where ρ', θ' vanish, which for the one-fold string is at $\sigma = \pi/2$. Therefore one has $\rho(\pi/2) = \rho_{\rm max}$, $\theta(\pi/2) = \theta_{\rm max}$. Demanding this, we get the conditions:

$$\frac{\pi}{2}\kappa\sin\alpha_0 = \int_0^{\theta_{\rm max}} d\theta \frac{1}{\sqrt{1 - a\sin^2\theta}} = \frac{\pi}{2\sqrt{a}}\ {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{1}{a}\right), \tag{2.12}$$

$$\frac{\pi}{2}\kappa\cos\alpha_0 = \int_0^{\rho_{\rm max}} d\rho \frac{1}{\sqrt{1 - b\sinh^2\rho}} = \frac{\pi}{2\sqrt{b}}\ {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, 1; -\frac{1}{b}\right). \tag{2.13}$$

The hypergeometric function is related to $\mathcal{K}(m)$, the complete elliptic integral of the first kind ($\mathcal{K}(m) = \mathcal{F}(\pi/2, m)$).

The energy, spin, and R-charge of the soliton are given by the following formulas:

$$E = \frac{R^2}{2\pi}\kappa\int_0^{2\pi} d\sigma\cosh^2\rho = \frac{2R^2}{\pi\cos\alpha_0}\int_0^{\rho_{\rm max}} d\rho\ \frac{\cosh^2\rho}{\sqrt{1 - b\sinh^2\rho}}, \tag{2.14}$$

$$S = \frac{R^2}{2\pi}\int_0^{2\pi} d\sigma\ \omega\sinh^2\rho = \frac{2R^2\omega}{\pi\kappa\cos\alpha_0}\int_0^{\rho_{\rm max}} d\rho\ \frac{\sinh^2\rho}{\sqrt{1 - b\sinh^2\rho}}, \tag{2.15}$$

$$J = \frac{R^2}{2\pi}\int_0^{2\pi} d\sigma\nu\sin^2\theta = \frac{2R^2\nu}{\pi\kappa\sin\alpha_0}\int_0^{\theta_{\rm max}} d\theta \frac{\sin^2\theta}{\sqrt{1 - a\sin^2\theta}}. \tag{2.16}$$

Computing the integrals, we find

$$E = E(a, b) = \frac{R^2}{\sqrt{b}\cos\alpha_0}{}_2F_1\left(-\frac{1}{2}, \frac{1}{2}, 1; -\frac{1}{b}\right), \tag{2.17}$$

$$S = S(a, b) = \frac{R^2}{2b^{3/2}\cos\alpha_0}\sqrt{1 + b\cos\alpha_0^2}\,{}_2F_1\left(\frac{1}{2}, \frac{3}{2}, 2; -\frac{1}{b}\right), \tag{2.18}$$

$$J = J(a) = \frac{R^2}{2a}{}_2F_1\left(\frac{1}{2}, \frac{3}{2}, 2; \frac{1}{a}\right), \tag{2.19}$$

$$\tan\alpha_0 = \sqrt{\frac{b}{a}}\frac{{}_2F_1(\frac{1}{2}, \frac{1}{2}, 1; \frac{1}{a})}{{}_2F_1(\frac{1}{2}, \frac{1}{2}, 1; -\frac{1}{b})}.$$

These formulas define parametrically $E = E(J, S)$.

Let us now derive explicit analytic formulas for $E = E(J, S)$ in four different regimes, according to the cases J or $S \gg \sqrt{\lambda}$, and J or $S \ll \sqrt{\lambda}$. In all cases, we will consider $J, S \gg 1$. The regimes of large spin S and large R-charge J (as compared to $\sqrt{\lambda}$) correspond to $b \ll 1$ and to $a \sim 1$ respectively, whereas small S and J correspond to $b \gg 1$ and to $a \gg 1$.

Define

$$I_1(b) = \int_0^{\rho_{\rm max}} d\rho \frac{1}{\sqrt{1 - b\sinh^2\rho}}, \qquad I_2(b) = \int_0^{\rho_{\rm max}} d\rho \frac{\sinh^2\rho}{\sqrt{1 - b\sinh^2\rho}},$$

$$I_3(a) = \int_0^{\theta_{\rm max}} d\theta \frac{1}{\sqrt{1 - a\sin^2\theta}}, \qquad I_4(a) = \int_0^{\theta_{\rm max}} d\theta \frac{\sin^2\theta}{\sqrt{1 - a\sin^2\theta}}.$$

The basic expansion formulas we need are

$$I_1(b) \cong -\frac{1}{2}\log b\,, \qquad I_2(b) \cong \frac{1}{b} + \frac{1}{4}\log b\,, \qquad \text{for} \quad b \ll 1\,, \tag{2.20}$$

$$I_1(b) \cong \frac{\pi}{2\sqrt{b}}\left(1 - \frac{1}{4b}\right), \qquad I_2(b) \cong \frac{\pi}{4b^{3/2}}\,, \qquad \text{for} \quad b \gg 1\,, \tag{2.21}$$

and

$$I_3(a) \cong -\frac{1}{2}\log\frac{(a-1)}{16}\,, \qquad I_4(a) \cong -\frac{1}{2}\log\frac{(a-1)}{16} - 1\,, \qquad \text{for} \quad a \cong 1\,, \tag{2.22}$$

$$I_3(a) \cong \frac{\pi}{2\sqrt{a}}\left(1 + \frac{1}{4a}\right), \qquad I_4(a) \cong \frac{\pi}{4a^{3/2}}\,, \qquad \text{for} \quad a \gg 1\,. \tag{2.23}$$

Let us now consider the different cases:

I) $a \gg 1$, $b \gg 1$, b/a fixed (short strings): This is the regime of small S and J with fixed J/S.

From eqs. (2.14), (2.15), (2.16), we get, to leading order,

$$E \cong R^2\sqrt{a^{-1} + b^{-1}}\,, \qquad S \cong \frac{R^2}{2b}\,, \qquad J \cong \frac{R^2}{2a}\,, \tag{2.24}$$

and $\tan\alpha_0 \cong \sqrt{J/S}$. Thus

$$E^2 \cong 2\sqrt{\lambda}(J+S)\,, \qquad \frac{J}{\sqrt{\lambda}} \ll 1\,, \qquad \frac{S}{\sqrt{\lambda}} \ll 1\,, \tag{2.25}$$

which is the usual Regge-type spectrum of string theory in flat spacetime. This is expected, since short strings do not feel the curvature of spacetime. In the case $J/S \to 0$, eq. (2.25) reduces to the case discussed in ref. [6]. Equation (2.25) differs from the formula $E^2 \cong J^2 + 2\sqrt{\lambda}\ S$ of [30]. The reason is the following: in the case of [30], the string is located at $\theta = \pi/2$; in the present case, the string is located near $\theta = 0,\ \rho = 0$, where there is a symmetry $J \leftrightarrow S$ due to the symmetry of the solution at small ρ, θ under $\phi \leftrightarrow \psi$. From the gauge theory point of view, the soliton of [30] and the present solution correspond to different operators when $J \neq 0$.

II) $a \sim 1$, $b \ll 1$: This is the case of large R-charge J and large spin S. Then the string is long with $\theta_{\max} \sim \pi/2$. Now eqs. (2.14)–(2.16) give

$$E \cong \frac{2R^2}{\pi\cos\alpha_0}\left(\frac{1}{b} - \frac{1}{4}\log b\right), \tag{2.26}$$

$$S \cong \frac{2R^2}{\pi\cos\alpha_0}\left(\frac{1}{b} + \frac{1}{4}\log b\right), \tag{2.27}$$

$$J \cong -\frac{R^2}{\pi}\log\frac{(a-1)}{16} - \frac{2R^2}{\pi}\,, \tag{2.28}$$

and $\tan\alpha_0 = \frac{\log\frac{(a-1)}{16}}{\log b}$. Thus we obtain

$$E - S = \frac{\sqrt{\lambda}}{\pi}\sqrt{\log^2 b + \left(\frac{\pi J}{\sqrt{\lambda}} + 2\right)^2}\,. \tag{2.29}$$

The parameter b is a function of S and J, determined by the transcendental equation

$$S = \frac{2\sqrt{\lambda}}{\pi b|\log b|}\sqrt{\log^2 b + \left(\frac{\pi J}{\sqrt{\lambda}} + 2\right)^2}\,. \tag{2.30}$$

For $J/\sqrt{\lambda} \ll \log S/\sqrt{\lambda}$, we have $b \cong \frac{2\sqrt{\lambda}}{\pi S}$. Thus we obtain

$$\begin{aligned} E - S &\cong \sqrt{\left(J + \frac{2\sqrt{\lambda}}{\pi}\right)^2 + \frac{\lambda}{\pi^2}\log^2\frac{\pi S}{2\sqrt{\lambda}}} \\ &\cong \frac{\sqrt{\lambda}}{\pi}\log\frac{\pi S}{2\sqrt{\lambda}} + \frac{\pi(J + 2\sqrt{\lambda}/\pi)^2}{2\sqrt{\lambda}\log\frac{\pi S}{2\sqrt{\lambda}}}\,, \end{aligned} \tag{2.31}$$

$$\frac{J}{\sqrt{\lambda}} \gg 1\,, \qquad \frac{S}{\sqrt{\lambda}} \gg 1\,, \qquad \frac{J}{\sqrt{\lambda}} \ll \log\frac{S}{\sqrt{\lambda}}\,.$$

Corrections to this formula are of order $J^2/\log^2\frac{S}{\sqrt{\lambda}}$.

For $J/\sqrt{\lambda} \gg \log\frac{S}{\sqrt{\lambda}}$, we have $-b\log b = 2J/S$, so that

$$E - S \cong J + \frac{2\sqrt{\lambda}}{\pi} + \frac{\lambda}{2\pi^2 J}\log^2\frac{2J}{S}\,, \tag{2.32}$$

$$\frac{J}{\sqrt{\lambda}} \gg 1\,, \qquad \frac{S}{\sqrt{\lambda}} \gg 1\,, \qquad \frac{J}{\sqrt{\lambda}} \gg \log\frac{S}{\sqrt{\lambda}}\,.$$

The formulas (2.31), (2.32) agree with the corresponding formulas in [30] for a rotating string fixed at $\theta = \pi/2$. The reason is that in this regime $a \sim 1$, most of the contribution to the R-charge comes from the region $\theta = \pi/2$. The only difference is the shift in J by $2\sqrt{\lambda}/\pi$, which represents the subleading correction to the large $J/\sqrt{\lambda}$ expansion. As mentioned above, the presence of this term reflects the fact that the string state we are considering is not the same as the state considered in [30].

III) $b \ll 1$, $a \gg 1$: In this case $J/\sqrt{\lambda} \ll 1$, $S/\sqrt{\lambda} \gg 1$; the string is long with θ being nearly fixed at $\theta \cong 0$. Now $\tan\alpha_0 = \frac{\pi}{\sqrt{a}\ |\log b|}$. Using eqs. (2.26), (2.27), and J as in (2.24), we obtain

$$E - S = \frac{\sqrt{\lambda}}{\pi}|\log b|\left(1 + \frac{\pi^2 J}{\sqrt{\lambda}\log^2 b}\right), \tag{2.33}$$

with $b \cong \frac{2\sqrt{\lambda}}{\pi S}$. Thus

$$E - S = \frac{\sqrt{\lambda}}{\pi}\log\frac{S}{\sqrt{\lambda}} + \frac{\pi J}{\log\frac{\pi S}{2\sqrt{\lambda}}}\,, \tag{2.34}$$

$$\frac{J}{\sqrt{\lambda}} \ll 1\,, \qquad \frac{S}{\sqrt{\lambda}} \gg 1\,.$$

This formula is new, and differs from the analog limit (small J, large S) in [30]. Again, the reason is that the small J string spins in the region of small θ, whereas in [30] is always fixed at $\theta = \pi/2$.

JHEP06(2002)038

IV) $b \gg 1$, $a \sim 1$. Then the string extends from $\theta = 0$ to $\theta_{\rm max} \sim \pi/2$ with ρ being nearly fixed at $\rho \cong 0$. Now $J/\sqrt{\lambda} \gg 1$ and $S/\sqrt{\lambda} \ll 1$. Using eqs. (2.14), (2.15), (2.16), we obtain

$$E = \frac{2\sqrt{\lambda}}{\pi} I_3 \sqrt{1 + \frac{I_1^2}{I_3^2}} \left(1 + \frac{I_2}{I_1}\right), \tag{2.35}$$

$$S = \frac{2\sqrt{\lambda}}{\pi} I_3 \frac{I_2}{I_1} \sqrt{1 + (1+b)\frac{I_1^2}{I_3^2}}, \tag{2.36}$$

$$J \cong \frac{2\sqrt{\lambda}}{\pi}(I_3 - 1). \tag{2.37}$$

Combining these equations, we find

$$E \cong J + \frac{2\sqrt{\lambda}}{\pi} + S + \frac{\lambda S}{2J^2}, \tag{2.38}$$
$$\frac{J}{\sqrt{\lambda}} \gg 1, \qquad \frac{S}{\sqrt{\lambda}} \ll 1.$$

For $S = 0$, eq. (2.38) reduces to the result (1.3) of [6]. Equation (2.38) can be compared with the similar formula found in [30],

$$E \cong J + S + \frac{\lambda S}{2J^2}. \tag{2.39}$$

Note that in the present case the correction $2\sqrt{\lambda}/\pi$ is important, being larger than the next terms.

V) General case: Using the general formulas eqs. (2.17)–(2.19), one can do a numerical plot of $E = E(S,J)$ in the general case. We find that $E = E(S,J)$ smoothly interpolates between the different regimes described above, with no surprises at intermediates regimes.

3. Discussion

The regime IV) is closely related to strings in the pp-wave background. Indeed, this background is obtained as the limiting geometry seen by a particle moving along the ψ direction with large momentum J, sitting near $\rho = 0$ and near $\theta = \pi/2$ [12, 4].[1] To see that (2.39) can be understood from the string spectrum in the pp wave background, we start with eq. (1.2), and consider string states with spin S associated with rotations in the plane 1-2. Let us assume that it is a state of the Regge trajectory, so that only $n = 1$ oscillators in the directions 1-2 are excited (e.g. by acting $S/2$ times with $a_{1+}^\dagger$, $\tilde{a}_{1+}^\dagger$, on the light-cone vacuum, where $a_{n+}^\dagger = \frac{1}{\sqrt{2}}(a_n^{1\dagger} - i a_n^{2\dagger})$). Then we have $N_n = S + N_n^T$, where N_n^T contains the oscillators corresponding to the other directions. Thus eq. (1.2) becomes

$$E = J + S\sqrt{1 + \frac{\lambda}{J^2}} + \sum_{n=-\infty}^{\infty} N_n^T \sqrt{1 + \frac{\lambda n^2}{J^2}}, \tag{3.1}$$

[1] Note that our definition of θ in (2.1) differs by $\theta \to \pi/2 - \theta$ from the notation of [4].

For $J \gg \sqrt{\lambda}$, this gives

$$E \cong J + S + \frac{\lambda S}{2J^2} + \sum_{n=-\infty}^{\infty} N_n^T \sqrt{1 + \frac{\lambda n^2}{J^2}}\,. \tag{3.2}$$

This agrees with eq. (2.39), up to the contribution of extra oscillators. This quantum part of the spectrum can be captured by a semiclassical quantization around the soliton [6, 30].

Given the correspondence between physical string states and gauge theory operators, there must be operators in the dual gauge theory with dimension given by

$$\Delta \cong J + S + \frac{\lambda S}{2J^2} + \cdots \tag{3.3}$$

Denoting by $\phi^1, \ldots, \phi^6$ the six scalars of $\mathcal{N} = 4$ Yang-Mills theory and $Z = \phi^5 + i\phi^6$, the unique single trace operator of bare dimension $\Delta = J$ is given by [4] $\mathrm{tr}[Z^J]$. The spin S can be introduced by adding covariant derivatives to this operator, i.e. replacing S factors Z by $D_i Z = \partial_i Z + [A_i, Z]$, $i = 1, 2$, adding phase factors of the form $\exp(2\pi i l/J)$, and summing over all possible insertion points (l is the position of D_i along the string of Z's). We propose that the resulting operator:

$$\mathcal{O}_{i_1 \ldots i_S} = \sum_{l_1, \ldots, l_s = 1}^{J} \frac{1}{\sqrt{J} N^{J/2}} \mathrm{tr}[\ldots Z D_{i_1} Z \ldots Z D_{i_S} Z \ldots] e^{\frac{2\pi i}{J}(l_1 + \ldots + l_S)}\,, \qquad S \ll J\,, \tag{3.4}$$

should be identified with the soliton of [30] (which is fixed at $\theta = \pi/2$). One of the sums can be performed by using the cyclic property of the trace. For $S \ll J$, this operator is almost BPS. The bare dimension is $\Delta = J + S$. The correction $\frac{\lambda S}{2J^2}$ arises by a one-loop calculation, by considering a subset of Feynman diagrams, as follows. Each insertion of D_i (corresponding to $a_{1+}^{\dagger}$ acting on the light-cone vacuum) is treated similarly as the insertions of ϕ^r operators computed in [4].[2] They give the contribution to the anomalous dimension

$$(\Delta - J)_1 = 1 + \frac{\lambda}{2J^2}\,,$$

so that, for S insertions, we get $\Delta - J = S + \frac{\lambda S}{2J^2}$, in agreement with eq. (3.3). This gives evidence that these operators are not decoupled in the large λ limit, with λ/J^2 fixed.

Let us now consider the string of section 2, which is stretched from $\theta = 0$ up to $\theta = \theta_{\mathrm{max}}$. Consider the case $S = 0$ and $J \gg \sqrt{\lambda}$. The correction to the energy by a shift $\frac{2\sqrt{\lambda}}{\pi}$ in (2.38) has a simple interpretation. For a string of large J which is not stretched in the θ direction, its energy is $E = J$. When this string is stretched from $\theta = 0$ up to $\theta = \pi/2$, its energy must increase in a quantity approximately given by $E \cong J + \mathrm{tension} \times \mathrm{length}$. Indeed, using (2.10), the energy (2.14) can also be written as

$$E = \frac{2\sqrt{\lambda}}{\pi \sin\alpha_0} \int_0^{\theta_{\mathrm{max}}} d\theta \, \frac{1}{\sqrt{1 - a\sin^2\theta}} \cong J + \frac{\sqrt{\lambda}}{2\pi}\, 4 \int_0^{\frac{\pi}{2}} d\theta \ \cos\theta\,, \tag{3.5}$$

where we have used (2.16) and $a \sim 1$. Thus we recover eq. (2.38) for $S = 0$, $E \cong J + \frac{2\sqrt{\lambda}}{\pi}$.

[2] In the pp wave background there is a symmetry under the exchange of the directions 1234 and 5678, which in the gauge theory corresponds to the exchanges of D_i with ϕ^r.

JHEP06(2002)038

The possible decay of the string of section 2 into BPS states should be suppressed by powers of the string coupling g_s. From the gauge theory point of view, such string should correspond to a highly excited local operator. This operator should be a linear combination of operators containing S insertions of D_i into $\mathrm{tr}[Z^J]$ — to account for the spin as above — and, in addition, insertions of various ϕ^r, $r = 1, 2, 3, 4$. It remains an interesting question to identify the corresponding operator.[3]

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JHEP06(2002)038